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Tachyon tube and supertube

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Abstract: We search for tubular solutions in unstable D3-brane. With critical electric field $E = 1$, solutions representing supertubes, which are supersymmetric bound states of fundamental strings, D0-branes, and a cylindrical D2-brane, are found and shown to exhibit BPS-like property. We also point out that boosting such a *tachyon tube* solution generates string flux winding around the tube, resulting in helical electric fluxes on the D2-brane. We also discuss issues related to fundamental string, absence of magnetic monopole, and finally more tachyon tubes with noncritical electric field.

Keywords: Brane Dynamics in Gauge Theories, Tachyon Condensation.

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1. Introduction

Supertube is a tubular D2-brane that preserves 1/4 of supersymmetry [1]–[3]. This requires presence of D0 and fundamental string charge, both realized as electromagnetic field on the D2 worldvolume. Alternatively, it may be considered as blow-up of F1-D0 composites obtained via angular momentum. One unusual characteristic of supertube is that, as classical solution, they come with huge degeneracy and can take many different form [2, 4, 5]. For instance, the cross section of a supertube need not be circular and in fact can even be noncompact [6].

The simplest of such supertubes involves a cylinder of D2 with uniform magnetic field B and uniform electric field E directed along the length of the cylinder [1]. Let us consider the field strength of type,

$$F = BR\, d\theta \wedge dz + E\, dt \wedge dz\,, \tag{1.1}$$

where R is the radius of the cylinder while θ is of period 2π. With Born-Infeld action, the physical electric flux, along z direction and per unit θ, is

$$\Pi_z = R\frac{E}{\sqrt{1-E^2+B^2}}\,, \tag{1.2}$$

and the energy density per unit z and unit θ is

$$\mathcal{E} = R\frac{1+B^2}{\sqrt{1-E^2+B^2}} = \sqrt{(\Pi_z^2+R^2)(1+B^2)}\,. \tag{1.3}$$

Note that the total electric flux is $2\pi\Pi_z$ while the net magnetic flux per unit z is $2\pi RB$.

This configuration is known to become 1/4 supersymmetric when the electric field E is taken to the critical value $E = 1$. Upon this, we have an inverse relationship between magnetic field and flux density,

$$\Pi_z = R\frac{1}{B}\,, \tag{1.4}$$

and the energy density decomposes into two pieces nicely

$$\mathcal{E} = R\left(\frac{1}{B} + B\right). \tag{1.5}$$

In this last expression, the first term represents the energy of fundamental strings and the second represents D0 energy. Tension of D2 itself disappeared completely.

One aspect of this state particularly relevant for this note is that one can reduce size of the supertube without affecting the fundamental string charge. Let us fix the total electric flux

$$n_{\mathrm{F1}} = 2\pi\Pi_z = \frac{2\pi R}{B}\,, \tag{1.6}$$

and take the small radius limit, where the energy per unit length becomes

$$2\pi\mathcal{E} = n_{\mathrm{F1}} + \frac{(2\pi R)^2}{n_{\mathrm{F1}}} = n_{\mathrm{F1}} + n_{\mathrm{F1}}B^2\,. \tag{1.7}$$

So as we decrease amount of magnetic flux on D2, the cylindrical configuration collapses to infinitesimal tubular configuration which entirely consists of the fundamental string charge.

It was recently suggested that in the system of unstable Dp-brane decay, fundamental string may be realized as a limiting configuration of tubular D$(p-1)$-branes with electric flux on it [7]. Question of fundamental strings remains one of more puzzling aspect in the study of unstable D-brane decay [8]–[13], and very recently possible roles of topological defects in this question began to be addressed [14, 15]. Approaching it from low energy dynamics, several nontrivial interaction between topological defects were isolated, and it was found that tachyonic kink configurations tend to attract nearby and parallel electric flux lines [16, 17] forming a bound state of D$(p-1)$ brane and fundamental strings. The idea is to curl up such a bound state into a cylindrical configuration with only one noncompact direction and let the compact part shrink away to zero size.

Specific form of such tubular solution was obtained from lifting [18] the Callan-Maldacena solution of D$(p-1)$ brane [19, 20] to that of tachyon effective action on unstable Dp-brane and concentrating on the spike part of the configuration. Such a combination involves vanishing radius of the tube, which makes it fairly singular. In this note, we consider the case of unstable D3 brane and consider a family of solutions representing supertube. These may be regarded as a thickened version of such tubular fundamental string charge carrier and may be used as regulated and stabilized solution.

The D$(p-1)$ branes are also typically represented by infinitesimally thin domain wall solutions [21]. Nevertheless, smooth kink solutions are known and, in the theory of tachyon, are represented by an infinite array of kinks and anti-kinks to be interpreted as D-brane-anti-D-brane pairs [22]–[24]. When the tachyon couples to Born-Infeld gauge field, spectra of regular extended objects become rich [23, 25]. Some of these smooth solutions are interpreted as thickened D-branes or bound states of D-brane and F1. Topological kink solution with orthogonal and uniform critical electric flux appears to enjoy BPS properties. For infinite array of kinks-anti-kinks and a topological kink with orthogonal and uniform electric flux, exact boundary states were also found [7]. Naturally, this prompts us to ask whether there exists a regulated and thickened version of supertube-like solutions, which of course should reproduce genuine supertube configurations in the zero thickness limit.

Section 2 will introduce the setup and solve for supertube-like configuration with "critical" electric field. With a specific choice of potential, we find an analytic and smooth solutions of coaxial array of tubes with D0 and fundamental string charge. Surprisingly, this family of solutions seemingly saturate a BPS-like energy formula, and behave much like supertubes, except the D2 worldvolume is thickened. Taking an infinitesimally thin limit of this solution, we find singular supertube solutions of arbitrary radius which match up with expected properties of supertubes precisely. We close the section with brief comments on relevance of these solutions in the context of fundamental string formation.

Section 3 addresses the question of turning on more electric fields and point out that boosting the tubular solution lengthwise will tilt the flux such that fundamental string appears winding helically along the tube. Section 4 considers a different class of ansatz involving Dirac monopole in gauge sector. We show that the only static solution of such kind involves strict vacuum $T=\infty$ so that the monopole-like configuration cannot lead to any physical effect. The appendix explores supertube ansatz with noncritical $E<1$ case. We close with summary.[1]

2. Supertubes from unstable D3-brane

We begin this section with introducing effective tachyon action for unstable D3-brane [18] and [27]–[31]

$$S=-\mathcal{T}_3\int d^4x\, V(T)\sqrt{-\det(g_{\mu\nu}+\partial_\mu T\partial_\nu T+F_{\mu\nu})}\,. \tag{2.1}$$

This sort of behavior with potential multiplying the kinetic term seems generic and in particular predicted by boundary string field theory approach [32, 33]. For most of this note, we will adopt a convenient form of $V(T)$ [34]–[36] and [22], which has been derived from open string theory [37] recently,

$$V(T)=\frac{1}{\cosh\left(T/T_0\right)}, \tag{2.2}$$

where T_0 is $\sqrt{2}$ for the non-BPS D-brane in the superstring and 2 for the bosonic string, in the string unit.

We are interested in tubular configurations embedded in flat D3-brane, and will work in the cylindrical coordinate system,

$$x^\mu=(t,x^i)=(t,z,r,\theta)\,,\qquad ds^2=-dt^2+dz^2+dr^2+r^2d\theta^2\,. \tag{2.3}$$

[1] As this work was finished, a related work discussing singular supertube solution appeared on the net [26].

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Both the tachyon field and the electromagnetic fields are assumed to depend only on the radial coordinate r, and we employ ansatz for the electromagnetic fields consistent with supertube solution

$$\begin{aligned} T &= T(r)\,, \\ F_{0r} &= F_{0\theta} = F_{r\theta} = F_{zr} = 0\,. \end{aligned} \tag{2.4}$$

With the assumption of time independence, behavior of E_z and $F_{\theta z}$ is determined entirely by the Bianchi identity, which gives

$$E_z = E, \qquad \frac{F_{\theta z}}{r} = B(r) = \frac{\alpha}{r}\,, \tag{2.5}$$

with constant E and α.[2] Since we have

$$-X \equiv -\det(g_{\mu\nu} + \partial_\mu T \partial_\nu T + F_{\mu\nu}) = \left[(1-E^2)r^2 + \alpha^2\right](1+T'^2)\,, \tag{2.6}$$

the low energy effective action simplifies drastically

$$-\mathcal{T}_3 \int dtdzd\theta \int dr\; V(T)\sqrt{(1-E^2)r^2+\alpha^2}\sqrt{1+T'^2}\,, \tag{2.7}$$

where the prime $'$ denotes differentiation with respect to the radial coordinate r.

2.1 Tachyon tube

Since we are primarily concerned with supertube-like solutions, we may as well take $E = 1$ for start and work with the action

$$-\alpha\mathcal{T}_3 \int dtdzd\theta \int dr\; V(T)\sqrt{1+T'^2}\,. \tag{2.8}$$

We will come back in later part of the note for more general solution. Then the effective action above maps to that of a simple mechanical system with conserved "energy" by imagining r as "time", which immediately gives us the following integral of motion,

$$\frac{V(T)}{\sqrt{1+T'^2}} = \frac{\beta\alpha}{\mathcal{T}_3}\,, \tag{2.9}$$

where we introduced an arbitrary integration constant β. Equivalently this integral of motion can be obtained from the energy-momentum conservation of the full action. Eq. (2.9) may be rewritten as

$$\frac{1}{2}T'^2 + U(T) = -\frac{1}{2}\,, \tag{2.10}$$

with

$$U(T) = -\frac{1}{2(\alpha\beta)^2}\left[\mathcal{T}_3 V(T)\right]^2\,. \tag{2.11}$$

Note that, in the limit of vanishing magnetic field $\alpha = 0$, the equation becomes $V(T) = 0$ and it allows only trivial vacuum solution $T(r) = \pm\infty$.

[2]This ansatz fails Bianchi identity $\nabla \cdot B = 0$ at origin.

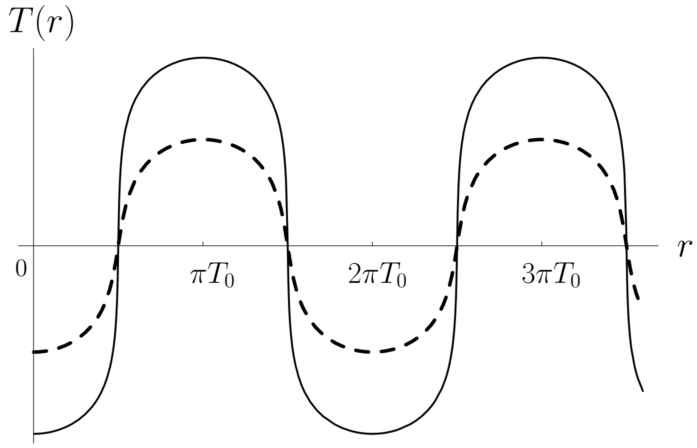

Figure 1: Plot of $T(r)$ with $r_0 = 0$, $T_0 = 1$, $\alpha = 1$, and $E = 1$: (i) solid line for $\alpha\beta/\mathcal{T}_3 = 0.001$, and (ii) dashed line for $\alpha\beta/\mathcal{T}_3 = 0.1$.

A nontrivial solution exists for $(\alpha\beta/\mathcal{T}_3)^2 < 1$ and is a coaxial array of tubular configuration where T oscillates as function of r. For instance, under the specific form of tachyon potential (2.2), we have an exact solution

$$T(r) = -T_0 \,\mathrm{arcsinh}\left[\sqrt{\left(\frac{\mathcal{T}_3}{\alpha\beta}\right)^2 - 1}\,\cos\left(\frac{r + r_0}{T_0}\right)\right]. \tag{2.12}$$

Note that this represents a coaxial array of tubular D2 and anti-D2 since the solution involves kink and anti-kink pair along every $\Delta r = 2\pi T$ [39]. For the moment, we shall impose regularity of T field at origin by requiring $r_0 = m\pi T_0$ where m is an integer. Note that the periodicity of the solution, $2\pi T_0$, is independent of the value of $(\alpha\beta/\mathcal{T}_3)^2$. This is a consequence of the particular form of the potential (2.2) we adopted.

For the obtained tubular objects, the following physical momenta vanish,

$$T_{0z} = 0\,, \qquad T_{0r} = 0\,, \tag{2.13}$$

and all off-diagonal components of stress-tensor vanish as would be expected of a stationary solution. All nonvanishing conserved densities are dictated by a single quantity

$$\Sigma(r) \equiv \frac{\mathcal{T}_3}{\alpha} V(T)\sqrt{1 + T'^2} = \beta(1 + T'^2) \tag{2.14}$$

which, for the particular form of the potential (2.2), has an explicit expression

$$\Sigma(r) \to \beta\left\{1 + \frac{\sin^2(r/T_0)}{\left[(\mathcal{T}_3/\alpha\beta)^2 - 1\right]^{-1} + \cos^2(r/T_0)}\right\}. \tag{2.15}$$

The energy and the electric flux in infinitesimal $r dr d\theta$ are written as

$$\begin{aligned} \mathcal{E}\, dr d\theta &= (r^2 + \alpha^2)\Sigma(r)\, dr d\theta\,, \\ \Pi\, dr d\theta &= r^2\Sigma(r)\, dr d\theta\,, \end{aligned} \tag{2.16}$$

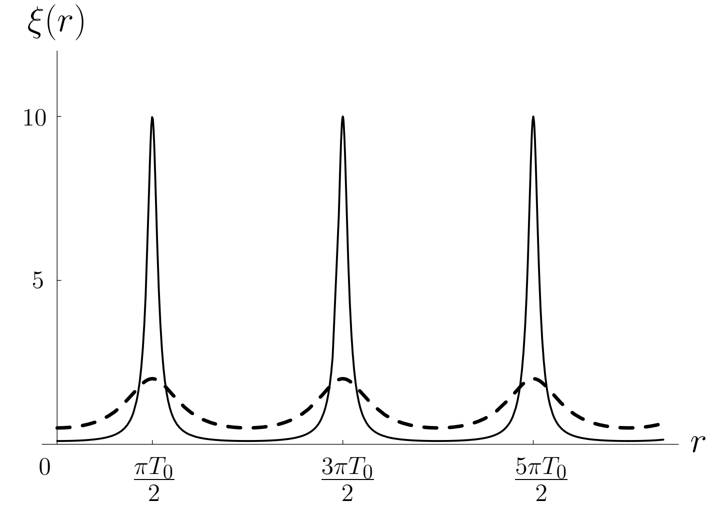

Figure 2: Plot of $\xi(r) \equiv \mathcal{E}(r) - \Pi(r)$ with $r_0 = 0$, $T_0 = 1$, $\alpha = 1$, and $E_z = 1$: (i) solid line for $\alpha\beta/\mathcal{T}_3 = 0.1$, and dashed line for $\alpha\beta/\mathcal{T}_3 = 0.5$.

per unit length along z. For these quantities, we can see that the period along r is actually half of that of $T(r)$. This can be understood by realizing that solution represents a coaxial array of tubular D2 and anti-D2 branes with worldvolume gauge fields turned on. While a single isolated D2 in vacuum is given by singular kink, it is also known that modification of boundary condition may thicken the kink into a smooth thickened solution [23]–[25]. Each period of T contains a pair of tubular domain wall and anti-domain wall, while the above conserved quantities are insensitive on orientation of the wall. Distinction between D2 and anti-D2 can be seen from gradient of T which generates RR charge of D2 branes [38, 39, 18].

In order to identify the obtained tubular objects in each half period $\Delta r = \pi T_0$, we compute the energy (tube tension) per unit tube length

$$\mathcal{E}_2^{(n)} = 2\pi \int_{(n-1)\pi T_0}^{n\pi T_0} dr\, \mathcal{E} = 2\pi\beta \int_{(n-1)\pi T_0}^{n\pi T_0} dr\, \frac{(\mathcal{T}_3/\alpha\beta)^2 (r^2 + \alpha^2)}{1 + [(\mathcal{T}_3/\alpha\beta)^2 - 1]\cos^2(r/T_0)}\,, \tag{2.17}$$

and string charge per unit length

$$Q_{\mathrm{F1}}^{(n)} = 2\pi \int_{(n-1)\pi T_0}^{n\pi T_0} dr\, \Pi = 2\pi\beta \int_{(n-1)\pi T_0}^{n\pi T_0} dr\, \frac{(\mathcal{T}_3/\alpha\beta)^2\ r^2}{1 + [(\mathcal{T}_3/\alpha\beta)^2 - 1]\cos^2(r/T_0)}\,. \tag{2.18}$$

In addition, each unit tube (or antitube) carries angular momentum per unit length

$$L^{(n)} = -2\pi\alpha\beta \int_{(n-1)\pi T_0}^{n\pi T_0} dr \frac{(\mathcal{T}_3/\alpha\beta)^2\ r^2}{1 + [(\mathcal{T}_3/\alpha\beta)^2 - 1]\cos^2(r/T_0)}\,. \tag{2.19}$$

A salient point is that the difference $\mathcal{E} - \Pi$ is quite simple and can be identified as D0-brane charge. In terms of formulae above, we define $Q_{\mathrm{D0}}^{(n)}$ as

$$\mathcal{E}_2^{(n)} = Q_{\mathrm{D0}}^{(n)} + Q_{\mathrm{F1}}^{(n)}\,. \tag{2.20}$$

For the specific form of the tachyon potential (2.2), it coincides exactly with the D0-brane charge before introducing tachyon profile

$$Q_{\rm D0}^{(n)} = 2\pi\beta\alpha^2 \int_{(n-1)\pi T_0}^{n\pi T_0} dr \, \frac{(\mathcal{T}_3/\alpha\beta)^2}{1+[(\mathcal{T}_3/\alpha\beta)^2-1]\cos^2(r/T_0)} = 2\pi^2\alpha T_0 \mathcal{T}_3 \,, \tag{2.21}$$

regardless of value of β. Thus, much like the case of supertube we introduced earlier, the tension of unit tachyon tube (or antitube) system decomposes linearly into two additive pieces.

Furthermore $Q_{\rm D0}$, as the notation suggests, happens to be the energy density expected of D0-branes. To see this, we need to note that

$$\mathcal{T}_2 = \pi T_0 \mathcal{T}_3 \tag{2.22}$$

is the quantity that becomes tension of the D2-brane in this low energy theory. On the other hand, $2\pi\alpha$ is the net magnetic flux on the D2 per unit z length. Product of the two generates D0 charge in spacetime [40].

It is not difficult to see that, the β-dependence should be absent in any solution that is supposed to represent a supertube. On the other hand, it happened here only because of the particular form of the potential (2.2) adopted above.[3] A priori, there seems to be no rationale why the integral that defines $Q_{\rm D0}^{(n)}$ (2.21) must be independent of β with more general form of potential. Dependence on β will result in an anomalous dependence on the electric flux density and ruin the BPS-like energy formula.

Thus, we must caution the reader against taking this class of smooth solution too seriously. Precise matching of the above tachyon tube properties and those expected of supertube is probably accident of the choice of the specific form of the potential. As we discuss in next subsection, the above line of derivation can be repeated for other more generic form of tachyon potential. In such general case, one must take the limit of infinitesimal thickness of the domain wall to find such a simple behavior. Nevertheless, as long as we are working with such a specific potential, this gives an interesting option for handling supertube-like configurations smoothly.

2.2 Thin supertube

When we take a limit of $\beta \to 0$, the configuration gets squeezed near $r = (n-1/2)\pi T_0$ for each positive integer n. Every localized piece becomes sharply peaked at each site of the tachyon tube (or antitube) so that

$$\lim_{\beta\to 0} \beta(1+T'^2) = \frac{\pi\mathcal{T}_3 T_0}{\alpha} \sum_{n=1}^{\infty} \delta\left(r - \left(n-\frac{1}{2}\right)\pi T_0\right) , \tag{2.23}$$

as shown in figure 2.

This singular form of solution also implies that value of T lies at $\pm\infty$ away from these centers, and neither the value of T' nor B enters the dynamics directly except for discrete positions of tube (antitube). In such limit, we have no reason whatsoever to insist upon

[3] This form of potntial has been derived but in a manner justifiable only in certain limit [37].

regularity at the origin. We thus have more general *singular* solution such that

$$\lim_{\beta\to 0}\beta(1+T'^2)=\frac{\pi\mathcal{T}_3T_0}{\alpha}\sum_{n=1}^{\infty}\delta\left(r+r_0-\left(n-\frac{1}{2}\right)\pi T_0\right). \tag{2.24}$$

Without loss of generality we may consider values of r_0 such that $\pi T_0/2 > r_0 \geq 0$. Correspondingly, we have

$$\mathcal{E}_2^{(n)} = \frac{2\pi}{\alpha}\left[\left(\left(n-\frac{1}{2}\right)\pi T_0 - r_0\right)^2+\alpha^2\right]\times\pi\mathcal{T}_3T_0\,, \tag{2.25}$$

$$Q_{\rm F1}^{(n)} = \frac{2\pi}{\alpha}\left(\left(n-\frac{1}{2}\right)\pi T_0 - r_0\right)^2\times\pi\mathcal{T}_3T_0\,, \tag{2.26}$$

$$Q_{\rm D0}^{(n)} = 2\pi\alpha\times\pi\mathcal{T}_3T_0\,, \tag{2.27}$$

and the accompanying angular momenta,

$$L^{(n)} = -2\pi\left(\left(n-\frac{1}{2}\right)\pi T_0 - r_0\right)^2\times\pi\mathcal{T}_3T_0\,. \tag{2.28}$$

Though distribution of the D0 charge and its magnetic field profile are singular, induced energy contribution is finite as shown in eq. (2.25). Note that eq. (2.20) and eq. (2.26) satisfy the relation between radial size of n-th tube $R_n = (n-1/2)\pi T_0 - r_0$ and charges such as $R_n = \sqrt{Q_{\rm F1}^{(n)}Q_{\rm D0}^{(n)}/2\pi^2T_0\mathcal{T}_3}$.

Different choice of the tachyon potential will modify the universal function $\Sigma(r)$, but the limiting form will again involve delta-functions. For the following discussion we will consider more general form of the potential, and retains the requirement that the potential decays exponentially $\sim e^{-|T|}$ asymptotically. Other requirements are that $V(T)$ is monotonically decreasing in $|T|$ and that $V(0)=1$.

For convenience, take a half period of $T(r)$ that solves

$$T'(r)=\sqrt{\left(\frac{\mathcal{T}_3V(T)}{\beta\alpha}\right)^2-1}\,. \tag{2.29}$$

$T'(r)$ then starts out with value zero and begins to increase slowly as long as $V(T)$ value remains exponentially small in T. Conversely when T' remains small $V(T)$ value must remain small also. Let us take any of one of half period such that $T(r)$ starts at large negative value $-T_{\rm max}$, where $V(\pm T_{\rm max}) = \beta\alpha/\mathcal{T}_3$ for the rest of discussion. So the half period is determined by the integral

$$\Delta r_{\rm half} = \int_{-T_{\rm max}}^{T_{\rm max}}\frac{dT}{\sqrt{(\mathcal{T}_3V(T)/\beta\alpha)^2-1}}\,. \tag{2.30}$$

At some points near the middle of this half period, T begins to approach $-\mathsf{T}_0$ where T_0 now refers to characteristic scale where $V(T)$ is of order 1. In this region $T'(r)$ is very large, achieving a maximum value at $r=R$ such that $T(R)=0$. This means that the interval in r where T is comparable to T_0 is very short.

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Recall that the function Σ is given by

$$\Sigma(r) = \frac{\mathcal{T}_3}{\alpha} V(T)\sqrt{1+T'^2}\,. \tag{2.31}$$

We wish to approximate this expression by

$$\Sigma(r) \sim \Sigma_0 = \frac{\mathcal{T}_3}{\alpha} V(T) T'\,, \tag{2.32}$$

for monotonic and increasing $T(r)$. Of course this is possible only if T' is very large. In the region where T' is of order one, the equation for T above tells us that we also must have,

$$\frac{\mathcal{T}_3 V(T)}{\beta\alpha} \sim 1\,, \tag{2.33}$$

meaning that $V(T)$ is still of order β. Since we are taking the limit of $\beta \to 0$, this allows us to use the approximate formula Σ_0 in place of Σ. Within this half period, assuming that the latter is finite, $\Sigma(r)$ reduces to

$$\lim_{\beta\to 0} \Sigma(r) \quad \to \quad \frac{1}{\alpha} \times \left(\mathcal{T}_3 \int V(T)\, dT\right) \times \delta(r-R). \tag{2.34}$$

We have used the fact that the support of Σ, more precisely the region of r where Σ remains larger than, say, $\sim \beta^{1/3}$ vanishes rapidly as $\sim \beta^{1/3}$ when we take $\beta \to 0$.

The quantity in the parenthesis is to be identified with ordinary BPS D2-brane tension $\mathcal{T}_2$ [21]. With this, D0 charge and F1 charge are respectively,

$$\begin{aligned} Q_{\mathrm{D0}} &= 2\pi\alpha\mathcal{T}_2\,, \\ Q_{\mathrm{F1}} &= \frac{2\pi R^2}{\alpha}\mathcal{T}_2\,, \end{aligned} \tag{2.35}$$

and the energy is the sum of the two

$$\mathcal{E}_2 = Q_{\mathrm{D0}} + Q_{\mathrm{F1}}\,. \tag{2.36}$$

In order to compare this to ordinary supertube, we need to restore the factor of D2-brane tension, $\mathcal{T}_2$, in energy formulae of section 1 and identify α with BR.

Therefore, the supertube can be understood as that in singular zero-thickness limit of the obtained unit tachyon tube (or antitube) solution. The whole configuration we obtained is interpreted as coaxial array of tube-antitube. Once we take $\beta = 0$ limit, on the other hand, each and every tube becomes a complete solution on its own.

2.3 Tachyon tube and tubular fundamental string

As we saw earlier in this note, supertube provides an interesting method of blowing up a long fundamental string. While this is not much of advantage in ordinary supersymmetric string theory context, it may prove more useful when we consider how fundamental string forms from decay of unstable D-branes.

One of more intriguing scenarios that emerged recently involve string flux sitting at top of tachyon potential. However, no such stable solution exists and one must take a double scaling limit of cylindrical domain wall to reach such a state. That is, take an infinitely

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thin domain wall with fluxes on it and then take infinitesimal radius limit of a cylinder thereof. The string flux is imagined to be embedded in this infinitesimal cylinder. Because of such a double scaling limit, the object is a bit difficult to handle.

A priori, it is not obvious whether, in the absence of $D(p-1)$ branes nearby, there is really any tangible difference between such configurations and a tight collection of string fluid [10] sitting at some homogeneous vacuum. As was point out in ref. [7], the dynamics of the two will follow exactly the same, Nambu-Goto-like, equation of motion [10, 11, 41] away from $D(p-1)$ brane. One motivation for the current work is to provide a setting where more precise question about such stringy configurations can be asked.

By adding magnetic field to the tubular configuration, we essentially obviated one of the two scalings, and found a stable string-like configuration of small but finite tubular radius. Magnetic field strength necessary scales with the radius of the tube, so one can take smaller and smaller size tube by taking small magnetic field limit. For specific form of the potential, furthermore, the unit tachyon tube before taking thin supertube limit provides additional control, given that it also satisfies a BPS-type relation (2.20) and is given by exact and smooth solution (2.12). Existence of such a static and smooth profile in closed form may come in handy, at least for the purpose of having a set of solutions that approaches infinitesimally small tubular solution.[4]

While we concentrated on cylinders with circular cross section, it is well known that a supertube can be deformed to have an arbitrary cross section [2, 4, 6]. In the present context, this results in huge degeneracy which gives at best arbitrarily shaped distribution of fundamental string charges, much as in the case of string fluid in real vacuum. As was also pointed out previously, therefore, this alternate picture of fundamental string is so far unable to resolve the usual degeneracy problem [9, 10, 17], or equivalently the lack of confinement mechanism, at the level of classical effective theory. It remains to be seen how and at what level this degeneracy is lifted [8, 9, 12, 7].

3. Boosted tachyon tube with helical string flux

One might ask whether more general ansatz leads to new kind of tubular solutions. One of more obvious variation we can try is to turn on more electromagnetic field. In fact there is a simple generalization of the ansatz that brings us back to an equally simple integrable equation for T. Consider turning on E_θ in addition to E_z

$$F = E_z dt \wedge dz + E_\theta dt \wedge d\theta + \tilde{\alpha} d\theta \wedge dz \,. \tag{3.1}$$

With static field strength of r dependence alone, Bianchi identity demands that both E_θ and E_z are constants.

Note that $E_\theta \neq 0$ will generate fundamental string charge along angular direction in addition to the one along z direction, so the electric flux will have a helical shape along

[4] This is not to say that these are a degenerate one-parameter family of solutions associated with a supertube. For smooth tube solution, the net electric flux in each unit tube depends on the parameter β while the D0 charge remains unchanged for the particular form of the potential. What remains unchanged is that the energy gets contribution from that of fundamental strings and D0's only.

the tube. More precisely, the electric flux along z and θ directions has a relation

$$\Pi_\theta = \frac{E_\theta}{E_z}\Pi_z \,, \tag{3.2}$$

where we define $\Pi^\theta \equiv \delta S/\delta E_\theta = \Pi_\theta/r^2$, and $\Pi^z \equiv \delta S/\delta E_z$ in $A_0 = 0$ gauge. The effective action for the tachyon field $T = T(r)$ becomes

$$-\mathcal{T}_3 \int dtdzd\theta \int dr \; V(T)\sqrt{(1-E_z^2)r^2 + \left(\tilde{\alpha}^2 - E_\theta^2\right)}\sqrt{1+T'^2}\,. \tag{3.3}$$

As before, this becomes integrable once we impose the condition of critical electric field, $E_z^2 = 1$, since the lagrangian will then have no explicit r-dependence. Furthermore it is easy to show that, within cylindrical symmetry ansatz, this is the only case where simple first order integral of motion is possible.

In fact, the form of the action is identical to the case with $E_\theta = 0$ above if we identify

$$\sqrt{\alpha^2} = \sqrt{\tilde{\alpha}^2 - E_\theta^2}\,. \tag{3.4}$$

Note in particular that sensible solution will appear only if $\tilde{\alpha}^2 > E_\theta^2$. One can understand this by realizing that the two cases are in fact related by a Lorentz boost along z direction. Starting with the simplest case,

$$F = E_z dt \wedge dz + \alpha d\theta \wedge dz \,, \tag{3.5}$$

one may boost this field strength along z with velocity v,

$$F = E_z dt \wedge dz + \frac{\alpha v}{\sqrt{1-v^2}}\, dt \wedge d\theta + \frac{\alpha}{\sqrt{1-v^2}}\, d\theta \wedge dz \,. \tag{3.6}$$

We then may identify

$$\tilde{\alpha} = \frac{\alpha}{\sqrt{1-v^2}}\,, \qquad E_\theta = \frac{\alpha v}{\sqrt{1-v^2}}\,, \tag{3.7}$$

which also gives eq. (3.4) completing the assertion that turning on E_θ is equivalent to boosting and appropriate rescaling of B. Of course T is a scalar field and so T' is also invariant under the Lorentz boost along z direction.

Emergence of winding electric flux on D2, or equivalently winding fundamental string charge, upon a boost may be surprising. After all, the same will happen even if the D2-brane were wound on a topological circle, which seems to say that boosting generated net conserved string charge along the topological circle direction. This unexpected effect may be understood once we realize that worldvolume magnetic field also couples NS-NS field $B_{\mu\nu}$. Although it does not generate net string charge, the worldvolume magnetic field generates a spacelike current associated with string charge.[5] All that happen is that the Lorentz boost tilts this spacelike current toward time direction a bit, which effectively generates density of string charge.

[5] We thank Kimyeong Lee on this point.

4. Is there a smooth magnetic monopole in an unstable D3?

With a minimal assumption on nontrivial field content, the effective action was shown above to be of the form,

$$\int dr\, V(T)R(r)\sqrt{1+T'^2} \tag{4.1}$$

with

$$R(r) = \sqrt{(1-E^2)r^2+\alpha^2} \tag{4.2}$$

for the supertube case with constant $rB = \alpha$. Reality of the Born-Infeld action requires, $E^2 \leq 1$, since otherwise the action will become purely imaginary at asymptotic region.

This sort of effective action can be used to explore another kind of configuration, namely a spherically symmetry magnetic monopole. The latter should have

$$F = Q\sin\theta d\theta\wedge d\phi \tag{4.3}$$

which gives an effective action of the above form with

$$R(r) = \sqrt{r^4+Q^2}\,. \tag{4.4}$$

Of course r in eq. (4.4) is the radial coordinate of spherical coordinate system.

Unlike the case of Maxwell theory, the singularity of this Dirac monopole does not cost infinite amount of energy, and one might wonder if there exist a Dirac monopole-like solution of finite mass. Existence of such a solution is unlikely, given that no obvious stringy interpretation is available. Here we will confirm this expectation by showing that the above class of action does not possess a regular and isolated solution that approaches vacuum at large radius.

If there is an isolated solution with such charges, $T(r)$ must approach $\pm\infty$ at large r. Since the system is invariant under $T\to -T$, take positive infinity for the asymptotic value of T. Far away from origin, then, the potential V approaches e^{-T} exponentially fast, and we could ask whether there is a proper solution with $V = e^{-T}$ and $T(\infty) = +\infty$. Euler-Lagrange equation of motion reduces to

$$\frac{d}{dr}\left(\frac{RT'}{\sqrt{1+T'^2}}\right) + \frac{R}{\sqrt{1+T'^2}} = 0\,, \tag{4.5}$$

where we used $\partial_T e^{-T} = -e^{-T}$. Defining f as

$$f(r) \equiv \frac{T'}{\sqrt{1+T'^2}}\,, \tag{4.6}$$

the equation of motion (4.5) is now

$$\frac{d}{dr}(Rf) + R\sqrt{1-f^2} = 0\,. \tag{4.7}$$

Because of the relative positive sign, this first order equation always drives f toward -1 from above as $r\to\infty$ for both choices of $R(r)$. Near $r=\infty$, we find

$$f = -1 + \frac{c}{2r^2} + \cdots\,, \tag{4.8}$$

where $c = 1$ for supertube ansatz (4.2) and $c = 4$ for monopole ansatz (4.4). The corresponding T is, on the other hand,

$$T(r) = \int dr \, \frac{f}{\sqrt{1-f^2}} = -\int dr \, \frac{r}{\sqrt{c}} + \cdots = -\frac{r^2}{2\sqrt{c}} + \cdots \tag{4.9}$$

which is inconsistent with original assumption $T \to +\infty$. Thus there exists no isolated supertube or monopole solution such that T at finite r is finite.

Thus all acceptable solutions that approach vacuum at infinity must be singular. In case of supertube ansatz, it corresponds to any one of the coaxial tubes in $\beta \to 0$ limit in section 2. In case of monopole, an obvious solution is found within above ansatz by setting $T = \infty$ everywhere or $T = -\infty$ everywhere. In this case, however, the Dirac monopole does not generate any measurable quantity, since all conserved quantum numbers we considered above vanish for this configuration. Only if there is an electric charge that couples to the gauge field directly (say, via minimal vector-current coupling), can there be physical consequence of this monopole solution. In the context of unstable D-brane, no such charged particle is known to exist.

What are other possible singular solutions? For instance, one logical possibility is to have a singular shell of domain wall at some finite radius, namely to have a spherical shell of a D2-brane. Due to the monopole-like gauge field, this D2 will have D0 charge spread uniformly on its worldvolume. However, we already know that this is not going to be stable. The reason can be found in Sen's recent construction of low energy dynamics of singular domain walls [21]. The latter is nothing but Born-Infeld action of the BPS D2-brane. Unlike the supertube case, where electric and magnetic fields on D2 can conspire to support the finite cylindrical structure, the case of monopole-like ansatz involves only magnetic field on worldvolume of the spherical D2. For Born-Infeld dynamics, it is always favorable to collapse such D2-shell to vanishing size.[6] Once this happens, the remaining object is nothing but point-like D0 charge sitting at origin.

5. Summary

In this note we considered static tubular solution in unstable D3-brane decay with an ansatz that involves both D0 charge and fundamental string charge. A series of both smooth and singular solutions are obtained and compared to supertube configuration of D2. While for a specific choice of potential there exists a smooth BPS-like solution, for generic choice of potential, one must take a singular limit to find supertube solution. After a brief comment on possible relevant of such solutions in study of fundamental string formation, we turned to other exotic solutions. Finally we show that smooth magnetic monopole solution does not exist as a sensible finite energy solution.

[6] See ref. [42] for related spherical solution. These solutions are smooth because of derivative expansion of the Born-Infeld action and also stabilized due to background RR-flux.

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A. Tachyon tubes with noncritical electric field

In the previous sections we dealt with tubular objects with critical value of the electric field $E^2 = 1$. In this appendix let us consider arbitrary value of the z-component of the electric field $E^2 < 1$ and investigate effect of it. As we saw above, isolated solution does not exist. Rather we shall find that the coaxial structure found in $E^2 = 1$ case still persist, but with some quantitative modification. Below we will study this case.

Finding solution is a matter of finding classical solution of the following reduced lagrangian

$$-V(T)\sqrt{(1-E^2)r^2+\alpha^2}\sqrt{1+T'^2}\,, \tag{A.1}$$

and we have the following second order equation to solve.

$$\frac{T''}{1+T'^2}+\frac{(1-E^2)rT'}{(1-E^2)r^2+\alpha^2}=\frac{1}{V}\frac{dV}{dT}\,. \tag{A.2}$$

Alternatively, we can get the same equation from r-component of energy-momentum conservation law, using

$$\begin{aligned} T_{rr} &= -\mathcal{T}_3\frac{V}{\sqrt{-X}\,r}\left[r^2(1-E^2)+\alpha^2\right], \\ T_{\theta\theta} &= -\mathcal{T}_3\frac{Vr^3}{\sqrt{-X}}(1+T'^2)(1-E^2)\,. \end{aligned} \tag{A.3}$$

Since we kept the orthogonality between electric and magnetic fields, physical electric flux is such that

$$\Pi_r=\Pi_\theta=0,\quad \Pi(\equiv\Pi_z)=\mathcal{T}_3\frac{Vr^2E}{\sqrt{-X}}(1+T'^2)\,, \tag{A.4}$$

where $-X$ is given in eq. (2.6). Π in eq. (A.4) contains a factor of $\sqrt{-X}$, so we can see that E^2 larger than 1 is not acceptable physically, if we wish to maintain reality of the action. Thus we will consider $E^2 < 1$ from now on.

With this, the second term of eq. (A.2) gives always damping effect. A natural boundary condition at the origin is again to assume $T'(0) = 0$. Then the damping term of eq. (A.2) does not affect choice of boundary value of the tachyon field, so it can have any value similar to the free parameter β in eq. (2.9) for the case of $E^2 = 1$, say $T(0) = -\bar{T}$.

Since the tachyon profile should be nonsingular, power series expansion near the origin gives

$$T(r) \approx -\bar{T} + a_1 r^2 + a_1 a_2 r^4 + \mathcal{O}(r^6) \tag{A.5}$$

with

$$a_1 = \frac{\tanh(\bar{T}/T_0)}{2T_0}, \tag{A.6}$$

$$a_2 = \left[\frac{\tanh(\bar{T}/T_0)}{2T_0}\right]^2 - \frac{1}{12T_0^2} - \frac{1-E^2}{6\alpha^2}. \tag{A.7}$$

Since the coefficient of r^2 term is determined only by the tachyon potential and double derivative term of eq. (A.2), it is independent of the electric and magnetic fields. Its signature is opposite to that of initial value $-\bar{T}$, which is consistent with monotonically increasing (decreasing) behavior of tachyon tube (antitube). In the limit of infinite $\bar{T}$, both second and third terms in eq. (A.5) remain finite and this implies possibility of zero thickness limit of the tachyon tube with keeping the period finite. As expected, electromagnetic contribution appears as decreased slope in the r^4 term (A.7) and it implies increment of the period (see figure 3). Even in the limit of vanishing $\alpha^2/(1-E^2)$ but not exactly zero, the coefficient of the second term of eq. (A.2) has $1/r$ which decreases as r increases. For sufficiently large r, the approximated equation does not afford the decaying solution to zero as $1/r^p$ form, so the critical or overdamping solution seems unlikely.

Near r_c with $T(r_c) = 0$, power series solution of eq. (A.2) is

$$T(r) \approx b_0(r - r_c)\left[1 + b_1(r - r_c) + b_2(r - r_c)^2 + \cdots\right] \tag{A.8}$$

with

$$b_1 = -\frac{r_c(1+b_0^2)}{2\left(r_c^2 + \frac{\alpha^2}{1-E^2}\right)}, \tag{A.9}$$

$$b_2 = \frac{1+b_0^2}{6}\left[-\frac{1}{T_0^2} - \frac{1}{r_c^2 + \frac{\alpha^2}{1-E^2}} + \frac{3r_c^2(1+b_0^2)}{\left(r_c^2 + \frac{\alpha^2}{1-E^2}\right)^2}\right], \tag{A.10}$$

where b_0 and r_c are determined by proper behavior near the origin. Existence of even order terms in eq. (A.8), e.g., nonvanishing b_1 (A.9), breaks reflection symmetry about r_c and stands for signal of the damping due to the noncritical electric field. In order to confirm the limit of step function for sufficiently large $\alpha^2/(1-E^2)$, let us compare the results (A.8) with the critical case (2.12). Expansion of $T(r)$ in eq. (2.12) near $r_{c0} = (2n \pm \frac{1}{2})\pi T_0$:

$$T(r) \approx \pm\sqrt{\left(\frac{\mathcal{T}_3}{\alpha\beta}\right)^2 - 1}\,(r - r_{c0})\left[1 - \frac{\mathcal{T}_3^2}{6T_0^2\alpha^2\beta^2}(r - r_{c0})^2\right] + \mathcal{O}((r - r_{c0})^5). \tag{A.11}$$

Taking the limit of critical electric field $E^2 \to 1$ in eq. (A.8) with assumption $r_c \to r_{c0}$, we observe as expected that

$$b_0 \overset{E^2\to 1}{\longrightarrow} \pm\sqrt{\left(\frac{\mathcal{T}_3}{\alpha\beta}\right)^2 - 1}, \qquad b_0 b_1 \overset{E^2\to 1}{\longrightarrow} 0. \tag{A.12}$$

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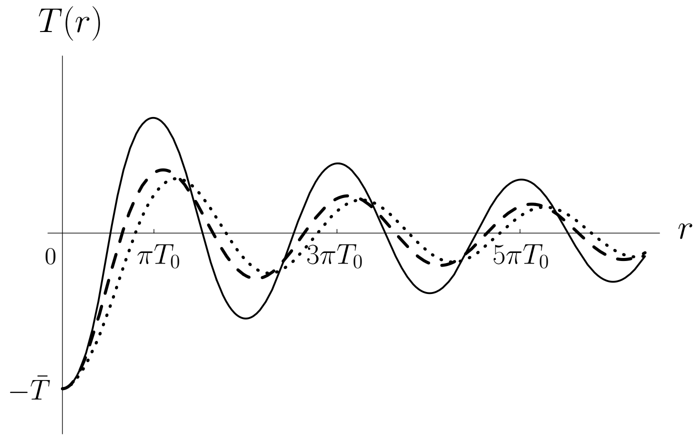

Figure 3: Plots of $T(r)$ with common initial value $-\bar{T}$: (i) solid line for $\alpha^2/(1-E^2) = 10$, (ii) dashed line for $\alpha^2/(1-E^2) = 1$, and (iii) dotted line for $\alpha^2/(1-E^2) = 0.1$.

The limiting value of $b_0 b_2$ consistently reproduces the cubic term in eq. (A.11). Therefore, near the critical E, we can safely take zero thickness limit in a consistent way with $\beta \to 0$ limit. On the other hand, once we take $b_0 \to \infty$ limit first with keeping $\alpha^2/(1-E^2)$ finite, then $b_0 b_1 \sim -r_c b_0^3 / \left[2(r_c^2 + \frac{\alpha^2}{1-E^2})\right]$ approaches $\mp\infty$ as far as r_c remains to be finite. Survival of even order terms implies asymmetry of T in the vicinity of r_c and then derivative term T'^2 is not expressed purely by δ-function.

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